Chapter 21: Electric Charge and Electric Field

Electric Charge

Ancient Greeks ~ 600 BC
Static electricity: electric charge via friction
(Attempted) pith ball demonstration:
  2 kinds of properties
  2 objects with same property repel each other
  2 objects with different properties attract each other
both properties are always created together

Benjamin Franklin:
  kinds of charges are positive and negative
  by convention, negative charge associated with amber

Conservation of Charge: The algebraic sum of all the electric charges in any closed system is constant.
Conductors and Insulators

(Objects are usually “charged” by moving electric charge around, rather than creating or destroying charge.)

Conductor:

charge passes easily through the material

=> conductors contain charges which are free to move

Insulator:

charge cannot move (easily) through material

Semiconductor:

transition between insulator and conductor, usually of interest because of exotic electrical properties
Charging by induction.
Quantization and Conservation of Charge

Microscopic structure of matter: Atoms

Nucleus

most of mass
positive charge
composed of protons (each has charge = +e) and neutrons (no electric charge)

“orbiting” electrons (each has charge = −e)

Atoms tend to be charge neutral

charge quantized: e = 1.6x10^{-19} C

Charge transfer usually in the form of addition or removal of electrons.
Coulomb’s Law

A description of the interaction between two (point) charges

The magnitude of the Force exerted by one charge on the other is proportional to the magnitude of each of the charges is inversely proportional to the square of the distance between the charges acts along a line connecting the charges

\[ F = k \frac{|Qq|}{r^2} \quad k = \frac{1}{4\pi \varepsilon_0} = 8.988 \times 10^9 \frac{N \cdot m^2}{C^2} \approx 9 \times 10^9 \frac{N \cdot m^2}{C^2} \]

Unit of charge is the Coulomb, a new type of quantity.

How big is 1 coulomb?
Another form of Coulomb’s Law

the force exerted by \( Q \) (“source charge”) on \( q \) (“test charge”)

\[
\vec{F} = k \frac{Qq}{r^2} \hat{r}
\]

\( \vec{F} \) = Force on \( q \)
\( \hat{r} \) = unit vector, from \( Q \) to \( q \)
\( r \) = distance, from \( Q \) to \( q \)
Electric Charge and Electric Field (cont’d)

Coulomb’s Law: the force exerted by $Q$ on $q$

$$\vec{F} = k \frac{Qq}{r^2} \hat{r}$$

For several Sources

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$

$$= k \frac{Q_1 q}{r_1^2} \hat{r}_1 + k \frac{Q_2 q}{r_2^2} \hat{r}_2 + \cdots$$

$$= \sum_i k \frac{Q_i q}{r_i^2} \hat{r}_i$$
• Analyze geometry/draw diagram
• calculate magnitude of each contribution  \( F_i = k \frac{|Q_iq|}{r_i^2} \)
• calculate components of each contribution
• add contributions as vectors (add component by component)

A charge \( q = 5.0 \text{ nC} \) is at the origin. A charge \( Q_1 = 2.0 \text{ nC} \) is located 2cm to the right on the x axis and \( Q_2 = -3.0 \text{ nC} \) is located 4 cm to the right on the x axis. What is the net force on \( q \)?
\( Q_1 = 2.0 \, \mu C \)

\( Q_2 = 2.0 \, \mu C \)

\( q = 4.0 \, \mu C \)

Geometry

Magnitudes

Components

Net Force
Example: Compare the electric repulsion of two electrons to their gravitational attraction

\[ F = k \frac{|Qq|}{r^2} \quad k \approx 9 \times 10^9 \frac{N \cdot m^2}{C^2} \]

\[ F = G \frac{Mm}{r^2} \quad G \approx 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \]
Electric Field and Electric forces

Electric field is a “disturbance” in space resulting from the presence of (source) charge, which exerts a force on a (test) charge.

\[ \vec{E}(\text{at } P) \equiv \frac{\vec{F}(\text{at } P)}{q} \]

\[ \vec{F}(\text{on } q) \equiv q\vec{E}(\text{at } q) \]
\[ \vec{E} = k \frac{Q}{r^2} \hat{r} \]

\( \vec{E} \) = Field at P
\( \hat{r} \) = unit vector, from Q to P
\( r \) = distance, from Q to P

\[ E = k \frac{|Q|}{r^2} \]
For several Sources

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots
\]

\[
= k \frac{Q_1}{r_1^2} \hat{r}_1 + k \frac{Q_2}{r_2^2} \hat{r}_2 + \cdots
\]

\[
= \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i
\]

Force Law: \( \vec{F} = q \vec{E} \)
Elementary Electric Field Examples:

- What is the electric field 30 cm from a +4nC charge?

- When a 100 volt battery is connected across two parallel conducting plates 1 cm apart, the resulting charge configuration produces a nearly uniform electric field of magnitude $E = 1.00 \times 10^4$ N/C.

- Compare the electric force on an electron in this field with its weight.
  
  What is the acceleration of the electron?
  
  The electron is released from rest at the top plate.
  
  What is the final speed of the electron as it hits the second plate?
  
  How long does it take the electron to travel this distance?
Electric Field Calculations

• Analyze geometry/draw diagram
  
  Electric Field Contributions are directed away from positive charges, toward negative charges

• calculate magnitude of each contribution  \( E_i = k \frac{|Q_i|}{r_i^2} \)

• calculate components of each contribution

• add contributions as vectors (add component by component)
Electric Dipole: two equal size($Q$), opposite sign charges separated by a distance ($l = 2a$). Determine the electric field on the x-axis.
Field of an electric dipole

\[ E_x = 0 \]

\[ E_y = -k \frac{2aQ}{(x^2 + a^2)^{3/2}} \]

\[ = -k \frac{p}{(x^2 + a^2)^{3/2}} \]

\[ \xrightarrow{x \gg a} -k \frac{p}{x^3} \]
Line of charge: uniform line of charge (charge $Q$, $l = 2a$ oriented along y-axis).

Determine the electric field on the x-axis

Geometry

\[
\frac{dQ}{Q} = \frac{dy}{2a} \quad \lambda = \frac{Q}{2a}
\]

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r}
\]

\[
r = \sqrt{x^2 + y^2}
\]

Magnitudes

\[
|d\vec{E}| = k \frac{dQ}{r^2} = k \frac{\lambda \; dy}{x^2 + y^2}
\]

Components

\[
dE_x = |d\vec{E}| \cos \theta = k \frac{\lambda \; dy}{x^2 + y^2} \frac{x}{\sqrt{x^2 + y^2}}
\]
Add Components

\[ E_x = \int_{\text{all charge}} dE_x \]

\[ = \int_{y=-a}^{y=a} k \frac{\lambda \, dy}{(x^2 + y^2)^{3/2}} \]

\[ = \frac{2k\lambda}{x\sqrt{x^2 + a^2}} \]

\[ = \frac{kQ}{x\sqrt{x^2 + a^2}} \]

\[ E_x \approx k \frac{Q}{x^2}, \quad x >> a \]

\[ E_x \approx \frac{2k\lambda}{x}, \quad x << a \]

infinite line of charge: \[ E_r = \frac{2k\lambda}{r} \]
Next: infinite sheet of charge is composed of a series of infinite lines of charges

Look carefully at related textbook examples of a ring of charge, and a disk of charge.
Electric Field due to an infinite sheet of charge

A sheet of charge composed of a series of infinite lines of charge.

$\sigma =$ charge per area

**Geometry**

$$\lambda = \sigma \, dz$$

$$\sin \theta = \frac{z}{r} \quad \cos \theta = \frac{y}{r}$$

$$r = \sqrt{z^2 + y^2}$$

**Magnitudes**

$$|d\vec{E}| = \frac{2k\lambda}{r} = \frac{2k\sigma \, dz}{\sqrt{z^2 + y^2}}$$

**Components**

$$dE_y = |d\vec{E}| \cos \theta$$

$$= \frac{2k\sigma \, dz}{\sqrt{z^2 + y^2}} \cdot \frac{y}{\sqrt{z^2 + y^2}}$$
Add Components

\[ E_y = \int dE_y \text{ all charge} \]

\[ = \int_{z=-\infty}^{z=\infty} \frac{2k\sigma y dz}{z^2 + y^2} \]

\[ = 2k\sigma y \frac{1}{y} \arctan \frac{z}{y} \bigg|_{z=-\infty}^{\infty} \]

\[ = 2k\sigma \left( \frac{\pi}{2} - \frac{-\pi}{2} \right) = 2k\sigma \pi \]

\[ = \frac{\sigma}{2\varepsilon_0} \left( k = \frac{1}{4\pi\varepsilon_0} \right) \]

uniform electric field!!
Interaction of an electric dipole with an electric field

dipole in a uniform electric field

\[ \sum \vec{F}_i = 0 \]

torque about center of dipole

\[ \vec{\tau} = \sum \vec{\tau}_i = \sum \vec{r}_i \times \vec{F}_i \quad \text{RHR!} \]

\[ \tau_z = \frac{l}{2} F \sin \theta + \frac{l}{2} F \sin \theta \]

\[ = l q E \sin \theta \]

\[ = p E \sin \theta \]

\[ \vec{\tau} = \vec{p} \times \vec{E} \]
Work done rotating dipole in an electric field

Watch directions of $\tau$, $\theta$ and $d\theta$

\[
dW = \tau \ d\theta = -pE \sin \theta \ d\theta
\]

\[
W = \int_{\theta_1}^{\theta_2} -pE \sin \theta \ d\theta
\]

\[
= pE \cos \theta_2 - pE \cos \theta_1 = -\Delta U
\]

\[
U(\theta) = -pE \cos \theta
\]

\[
= -\vec{p} \cdot \vec{E}
\]
“Lines of Force”

Electric Field Lines: a means of visualizing the electric field

=A line in space always tangent to the electric field at each point in space
concentration give indication of field strength
direction give direction of electric field
• start on positive charges, end on negative charges
• electric field lines never cross