Ch26: Direct Current Circuits

Two Basic Principles:
- Conservation of Charge
- Conservation of Energy

Resistance Networks

\[ V_{ab} = IR_{eq} \]
\[ R_{eq} \equiv \frac{V_{ab}}{I} \]
Resistors in series

Conservation of Charge
\[ I = I_1 = I_2 = I_3 \]

Conservation of Energy
\[ V_{ab} = V_1 + V_2 + V_3 \]

\[ R_{eq} \equiv \frac{V_{ab}}{I} = \frac{V_1 + V_2 + V_3}{I} \]
\[ = \frac{V_1}{I} + \frac{V_2}{I} + \frac{V_3}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} \]
\[ R_{eq} = R_1 + R_2 + R_3 \]

Voltage Divider:
\[ \frac{V_1}{V_{ab}} = \frac{R_1}{R_{eq}} \]
Resistors in parallel

Conservation of Charge

\[ I = I_1 + I_2 + I_3 \]

Conservation of Energy

\[ V_{ab} = V_1 = V_2 = V_3 \]

\[ \frac{1}{R_{eq}} = \frac{I}{V_{ab}} = \frac{I_1}{V_1} = \frac{I_2}{V_2} = \frac{I_3}{V_3} \]

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

Current Divider:

\[ \frac{I_1}{I} = \frac{R_{eq}}{R_1} \]
Example:
Determine the equivalent resistance of the circuit as shown.
Determine the voltage across and current through each resistor.
Determine the power dissipated in each resistor
Determine the power delivered by the battery
Kirchoff’s Rules

Some circuits cannot be represented in terms of series and parallel combinations.

Kirchoff’s rules are based upon conservation of energy and conservation of charge.
Conservation of Charge: Junction rule

The algebraic sum of the currents into any junction is zero.

\[ \sum I = 0 \]
\[ = + I_1 + I_2 - I_3 \]

Alternatively: The sum of the currents into a junction is equal to the sum of the currents out of that junction.

\[ I_1 + I_2 = I_3 \]
Conservation of Energy: Loop rule

The algebraic sum of the potential differences around any closed loop is zero.
Potential Differences in the direction of travel

\[ V = -\mathcal{E} \]

\[ V = +\mathcal{E} \]

\[ V = -IR \]

\[ V = +IR \]
\[ +I_1 + I_2 - I_3 = 0 \]
\[ -\mathcal{E}_1 + I_1 R_1 - I_2 R_2 + \mathcal{E}_2 = 0 \]
\[ -I_3 R_3 - I_2 R_2 + \mathcal{E}_2 = 0 \]
\[ -I_3 R_3 - I_1 R_1 + \mathcal{E}_1 = 0 \]
\[ + I_1 + I_2 - I_3 = 0 \]
\[ -12 + I_1 2 - I_2 1 + 5 = 0 \]
\[ -I_3 3 - I_2 1 + 5 = 0 \]
Standard linear form:

+ $I_1$ + $I_2$ − $I_3$ = 0
− 12 + $I_1$ 2 − $I_2$ $I_1$ + 5 = 0
− $I_3$ 3 − $I_2$ $I_2$ + 5 = 0

TI-89: [2nd] [MATH] [4](selects matrix) [5] (selects simult)

Entry line: simult([1,1,-1;2,-1,0;0,1,3],[0;7;5])

$I_1$ = 3
$I_2$ = −1
$I_3$ = 2

or (using [F2])

solve(i1+i2-i3=0 and -12+2*i1-i2+5=0 and -3*i3-i2+5=0, {i1,i2,i3})

yields $i_1$ = 3 and $i_2$ = −1 and $i_3$ = 2
<table>
<thead>
<tr>
<th></th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>3A</td>
<td>(-)1A</td>
<td>2A</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\mathbf{P} & = \mathbf{E}_1 \\
\mathbf{P} & = \mathbf{E}_2 \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{E}_1 & = 12\text{V} \\
R_1 & = 2\Omega \\
\mathbf{E}_2 & = 5\text{V} \\
R_2 & = 1\Omega \\
R_3 & = 3\Omega \\
I_1 & \\
I_2 & \\
I_3 & \\
\end{align*}
\]
Electrical Instruments

Galvanometer:
- Torque proportional to current $I$
- Restoring torque proportional to angle

$\Rightarrow$ (Equilibrium) angle proportional to angle $\theta$

Maximum Deflection = “Full Scale Deflection”

- $I_{fs} = \text{Current to produce Full Scale Deflection}$
- $R_c = \text{Resistance (of coil)}$

$V_{fs} = I_{fs} R_c$

Example: $I_{fs} = 1.0 \, mA$, $R_c = 20.0 \, \Omega$

$V_{fs} = 20 \, mV = 0.020 \, V$
Wheatstone Bridge

Circuit Balanced: $V_{ab} = 0$

$I_G = 0$

$I_1 = I_3$ \hspace{0.5cm} $I_2 = I_4$

$V_1 = V_2$ \hspace{0.5cm} $V_3 = V_4$

\[
\frac{V_1}{V_3} = \frac{V_2}{V_4} \implies \frac{I_1R_1}{I_3R_3} = \frac{I_2R_2}{I_4R_4} \implies
\]

\[
\frac{R_1}{R_3} = \frac{R_2}{R_4}
\]
Ammeter

Measures current through device

Measure currents larger than \( I_{fs} \) by bypassing the galvanometer with another resistor (shunt Resistor \( R_s \)).

\[
I_a = \text{ammeter full scale}
\]

When \( I = I_a \), we need \( I_c = I_{fs} \)

\[
V_G = V_{sh} \Rightarrow I_{fs} R_c = (I_a - I_{fs}) R_{sh}
\]

\[
\frac{\theta}{\theta_{fs}} = \frac{I}{I_a}
\]

Design a 50 mA ammeter from example galvanometer

\( I_{fs} = 1.0 \text{ mA}, R_c = 20.0 \Omega, V_{fs} = 20 \text{ mV} = .020 V \)
Voltmeter

Measures potential difference across device

Measure \( V \) larger than \( V_{fs} \) by adding a resistor (\( R_s \)) in series to the galvanometer.

\[
V_m = \text{Voltmeter full scale}
\]

When \( V = V_m \), we need \( I_c = I_{fs} \)

\[
I_c = I_s, \quad V_m = V_s + V_G
\]

\[
\Rightarrow V_m = I_{fs} R_s + I_{fs} R_c
\]

\[
R_s = \left( \frac{V_m}{I_c} \right) - R
\]

\[
\frac{\theta}{\theta_{fs}} = \frac{I}{I_{fs}} = \frac{V}{V_m}
\]

Design a 10 V voltmeter from example galvanometer

\( I_{fs} = 1.0 \text{ mA}, \quad R_c = 20.0 \text{ }\Omega, \quad V_{fs} = 20 \text{ mV} = 0.020 \text{ V} \)
Ohmmeter

Measures resistance across terminals

Meter supplies an EMF, resistance is determined by response.

When $R = 0$, we need $I_c = I_{fs}$

\[ \mathcal{E} = (I_{fs} R_s + I_{fs} R_c) \]

\[ \Rightarrow V_m = I_{fs} R_s + I_{fs} R_c \]

\[ R_s = \left( \frac{\mathcal{E}}{I_{fs}} \right) - R_c \]

\[ \frac{\theta}{I_{fs}} = \frac{I}{R_s + R_c} = \frac{R_s + R_c}{R_s + R_c + R} \]
Resistance Capacitance Circuits

Charging Capacitor

Capacitor is initially uncharged.
Switch is closed at \( t=0 \)
Apply Loop rule:
\[ \mathcal{E} - \frac{q}{C} - iR = 0 \]
\[ \mathcal{E} - \frac{q}{C} - iR = 0 \]

\[ i = \frac{\mathcal{E}}{R} - \frac{q}{RC} = \frac{dq}{dt} \]

\[ \frac{dq}{dt} = -\frac{1}{RC}(q - \mathcal{E}C) \]

\[ \frac{dq}{(q - \mathcal{E}C)} = -\frac{1}{RC} dt \]

\[ \int_{0}^{q(t)} \frac{dq}{(q - \mathcal{E}C)} = \int_{0}^{t} -\frac{1}{RC} dt \quad u = q - \mathcal{E}C, \quad \int_{u}^{1} \frac{1}{u} du = \ln u \]

\[ \ln \left( \frac{q(t) - \mathcal{E}C}{-\mathcal{E}C} \right) = -\frac{1}{RC} t \]
\[
\begin{align*}
\ln\left(\frac{q(t) - \mathcal{E}C}{-\mathcal{E}C}\right) &= -\frac{1}{RC}t \\
q(t) &= \mathcal{E}C(1 - e^{-t/RC}) \\
&= Q_{\text{final}}(1 - e^{-t/\tau}) \\
\tau &= RC = \text{"TimeConstant"}
\end{align*}
\]

\[
i = \frac{dq}{dt} = \mathcal{E}C\left(\frac{1}{RC}e^{-t/RC}\right) = \frac{\mathcal{E}}{R}e^{-t/RC} = I_o e^{-t/\tau}
\]
Resistance Capacitance Circuits

Discharging Capacitor

Switch has an initial charge $Q_0$.
Switch is closed at $t=0$
Apply Loop rule:

$$+ \frac{q}{C} - iR = 0$$
\[ + \frac{q}{C} - iR = 0 \]

\[ i = \frac{q}{RC} = - \frac{dq}{dt} \]

\[ \frac{dq}{dt} = - \frac{1}{RC} q \]

\[ \frac{dq}{q} = - \frac{1}{RC} dt \]

\[ \int_{q_0}^{q(t)} \frac{dq}{q} = \int_{0}^{t} - \frac{1}{RC} dt \]

\[ \ln \left( \frac{q(t)}{Q_0} \right) = - \frac{1}{RC} t \]
\[ \ln \frac{q(t)}{Q_o} = -\frac{1}{RC} t \]

\[ q(t) = Q_o e^{-t/RC} \]

\[ = Q_o e^{-t/\tau} \]

\[ \tau = RC = \text{same "Time Constant"} \]

\[ i = -\frac{dq}{dt} = \frac{Q_o}{RC} e^{-t/RC} \]

\[ = I_o e^{-t/\tau} \]