Chapter 27: Magnetic Field and Magnetic Forces

Iron ore found near Magnesia

Compass needles align N-S: magnetic Poles
   North (South) Poles attracted to geographic North (South)
   Like Poles repel, Opposites Attract
   No Magnetic Monopoles

Magnetic Field Lines = direction of compass deflection.

Electric Currents produce deflections in compass direction.

=> Unification of Electricity and Magnetism in Maxwell’s Equations.
Magnetic Fields in analogy with Electric Fields

Electric Field:
- Distribution of charge creates an electric field $E(r)$ in the surrounding space.
- Field exerts a force $F = q E(r)$ on a charge $q$ at $r$

Magnetic Field:
- Moving charge or current creates a magnetic field $B(r)$ in the surrounding space.
- Field exerts a force $F$ on a charge moving $q$ at $r$
- (emphasis this chapter is on force law)
Magnetic Fields and Magnetic Forces

Magnetic Force on a moving charge
- proportional to electric charge
- perpendicular to velocity $\mathbf{v}$
- proportional to speed $v$ (for a given geometry)
- perpendicular to Magnetic Field $\mathbf{B}$
- proportional to field strength $B$ (for a given geometry)

$$\mathbf{F} = q \mathbf{v} \times \mathbf{B}$$
\[ F = q \mathbf{v} \times \mathbf{B} \]
\[ F = |q| \mathbf{v} \mathbf{B} \sin \theta \]
\[ = |q| \mathbf{v} \mathbf{B} \quad (\mathbf{v} \perp \mathbf{B}) \]

\[ F = q \mathbf{v} \times \mathbf{B} \]
\[ F = |q| \mathbf{v}_\perp \mathbf{B} \]

\[ F = q \mathbf{v} \times \mathbf{B} \]
\[ F = |q| \mathbf{v} \mathbf{B}_\perp \]
Magnetic Fields

Units of Magnetic Field Strength:

\[ [B] = \frac{[F]}{[q][v]} = \frac{N}{(C \cdot m \cdot s^{-1})} = \text{Tesla} \]

Defined in terms of force on standard current

CGS Unit 1 Gauss = 10^{-4} \text{ Tesla}

Earth's field strength \sim 1 \text{ Gauss}

Direction = direction of velocity which generates no force

Electromagnetic Force:

\[ F = q \left( E + v \times B \right) = \text{Lorentz Force Law} \]
Magnetic Field Lines and Magnetic Flux

Magnetic Field Lines
  Mapped out with compass
  Are not lines of force ($\mathbf{F}$ is not parallel to $\mathbf{B}$)
  Field Lines never intersect

Magnetic Flux

\[ d\Phi_B = \mathbf{B} \cdot d\mathbf{A} \]
\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]
\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{no magnetic charge! (no monopoles)} \]
• **SI Unit of Flux:**
  
  - 1 Weber = 1 Tesla x 1 m^2
  
  - for a small area \( B = \frac{d\Phi_B}{dA_\perp} \)
  
  - \( B \) = “Magnetic Flux Density”

  Flux through an open surface will play an important role
Motion of Charged Particles in a Magnetic Field

Charged Particle moving perpendicular to the Magnetic Field

- Circular Motion!
- (simulations)
Charged Particle moving perpendicular to a uniform Magnetic Field

\[ F = |q| v B = \frac{mv^2}{R} \]

\[ R = \frac{mv}{|q| B} \]

\[ \omega = \frac{v}{R} = \frac{|q| B}{m} \]

= cyclotron frequency
In a non-uniform field: Magnetic Mirror

Net component of force away from concentration of field lines.

Magnetic Bottle

Van Allen Radiation Belts
Work done by the Magnetic Field on a free particle:

\[ dW = \vec{F} \cdot d\vec{x} \]

\[ = \left( q\vec{v} \times \vec{B} \right) \cdot \vec{v} dt \]

\[ = 0! \]

=> no change in Kinetic Energy!
Motion of a free charged particle in any magnetic field has constant speed.
Applications of Charged Particle Motion in a Magnetic Field

Recall:

Charged Particle moving perpendicular to a uniform Magnetic Field

\[ F = |q|vB = \frac{mv^2}{R} \]

\[ R = \frac{mv}{|q|B} \]
Velocity Selector
makes use of crossed \( E \) and \( B \) to provide opposing forces

\[
\text{upwards} \quad F = q \vec{v} \times \vec{B} \\
\text{downwards} \quad F = qE
\]

No net deflection \( \Rightarrow \) forces exactly cancel:

\[
|q| \vec{v} \vec{B} = |q| \vec{E} \\
\vec{v} = \vec{E}/\vec{B}
\]
J. J. Thomson’s Measurement of $e/m$

Electron Gun

and velocity selector:

\[ v = \frac{E}{B} \]

\[ \frac{1}{2}mv^2 = eV \]

\[ \frac{e}{m} = \frac{E^2}{2VB^2} \]

\[ e/m = 1.76 \times 10^{11} \text{ C/kg} \]

with Millikan’s measurement of $e$

$\Rightarrow$ mass of electron
Example: Using an accelerating Potential of 150 V and a transverse Electric Field of $6 \times 10^6$ N/C. Determine

a) the speed of the electrons,

b) the magnetic field magnitude required for no net deflection
One method
velocity selector + circular trajectory

\[ v = \frac{E}{B} \]
\[ R = \frac{mv}{qB} \]
\[ m = \frac{RqB}{v} = \frac{RqB^2}{E} \]
Example: Vacuum System Leak Detector uses Helium atoms. Ionized helium atoms (He $^+$) are detected with a mass spectrometer with a magnetic field strength of .1 T. With a velocity selector tuned to $1 \times 10^5$ m/s, where must the detector be placed to detect $^4$He $^+$ ions?
Magnetic Force on a Current Carrying Wire

\[ F = \sum F_i = \sum q_i \vec{v}_i \times \vec{B} \]

\[ = Nq \vec{v}_d \times \vec{B} = n \cdot \text{volume} \cdot q \vec{v}_d \times \vec{B} \]

\[ = nA dl \vec{v}_d \times \vec{B} = \mathcal{J} A dl \times \vec{B} \]

\[ = I d\vec{l} \times \vec{B} \quad (RHR) \]
Example: A 1-m bar carries 50 A from west to east in a 1.2 T field directed 45° North of East. What is the magnetic force on the bar?

Force will be directed upwards (out of the plane of the page)

\[ F = ILB \sin \theta \]

\[ = 50A \times 1m \times 1.2T \times \sin 45° \]

\[ = 42.4N \]
Torque on a Current Loop (from $\mathbf{F} = I \mathbf{l} \times \mathbf{B}$)

Rectangular loop in a magnetic field (directed along z axis) short side length a, long side length b, tilted with short sides at an angle with respect to $\mathbf{B}$, long sides still perpendicular to $\mathbf{B}$.

Forces on short sides cancel: no net force or torque.
Forces on long sides cancel for no net force but there is a net torque.
Torque calculation: Side view

\[ \tau = F_b \frac{a}{2} \sin \theta + F_b \frac{a}{2} \sin \theta \]
\[ = I_{ab} B \sin \theta = I A B \sin \theta \]
\[ = I A \times B = \mu \times B \]

Magnetic Dipole
acts similarly to Electric Dipole

\[ U = -\mu \cdot B \]

Switch current direction every 1/2 rotation => DC motor
Hall Effect

Conductor in a uniform magnetic field

Magnetic force on charge carriers $F = q \mathbf{v}_d \times \mathbf{B}$

$F_z = q\mathbf{v}_d B$ Charge accumulates on edges
Equilibrium: Magnetic Force = Electric Force on bulk charge carriers

Charge accumulates on edges $F_z = 0 = qv_d B_y + q E_z$

$v_d = -\frac{E_z}{B_y}$

$J_x = n q v_d = -n q \frac{E_z}{B_y}$

Hall EMF $V_H = E_z w$

$I = J_x w$}

$nq = \frac{-IB_y}{V_H t}$

$p212c27: 23$
Negative Charge carriers:

velocity in negative x direction

magnetic force in positive z direction

\[ \Rightarrow \text{resulting electric field has reversed polarity} \]
Example: A ribbon of copper 2.0 mm thick and 1.5 cm wide carries a 75 A current in a .40 T magnetic Field. The resulting Hall emf is .81 μV. What is the density of charge carrying electrons?