Chapter 29: Electromagnetic Induction

Motional EMF’s

Conductor Moving through a magnetic field

Magnetic Force on charge carriers

Accumulation of charge

Balanced Electrostatic/Magnetic Forces

\[ qvB = qE \]

Induced Potential Difference

\[ \mathcal{E} = EL = vBL \]

Generalized

\[ d \mathcal{E} = v \times B \cdot dl \]
With conduction rails:

\[ F = qv \times B \]

demonstrations with galvanometer
moving conductor
moving field!
Consider circuit at right with total circuit resistance of $5 \, \Omega$, $B = 0.2 \, \text{T}$, speed of conductor = $10 \, \text{m/s}$, length of $10 \, \text{cm}$. Determine:
(a) EMF,
(b) current,
(c) power dissipated,
(d) magnetic force on conducting bar,
(e) mechanical force needed to maintain motion and
(f) mechanical power necessary to maintain motion.

\[
d\mathcal{E} = v \times B \cdot dl
\]
Faraday disk dynamo = DC generator

d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}

= \omega \ rBdr

\mathcal{E} = \int_0^R \omega \ rBdr

= \omega \ B \frac{R^2}{2}
Faraday’s Law of Induction

Changing magnetic flux induces and EMF

\[ d\Phi_B = \vec{B} \cdot d\vec{A} = B dA \cos \phi \]

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

\[ \Phi_B = \vec{B} \cdot \vec{A} = BA \cos \phi \text{ for a uniform magnetic field} \]

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -N \frac{d\Phi_B}{dt} \text{ for N loops} \]

**Lenz’s Law:** The direction of any magnetic induction effect (induced current) is such as to oppose the cause producing it.

(opposing change = inertia!)
Conducting rails: changing flux from changing area

\[ d\Phi = BdA = Blvd t \]

\[ \mathcal{E} = -\frac{d\Phi}{dt} = -Blv \]

watch sign conventions + RHR
Lenz's Law + induced currents
Simple Alternator

\[ \Phi_B = AB \cos \omega t \]

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} \]
\[ = -AB(-\omega \sin \omega t) \]
\[ = \omega AB \sin \omega t \]

\[ I = \frac{\mathcal{E}}{R} = \frac{\omega AB \sin \omega t}{R} \]

\[ P_{elec} = I^2R = \frac{(\omega AB \sin \omega t)^2}{R} \]

\[ M = IA = \frac{\omega A^2B \sin \omega t}{R} \]

\[ P_{mech} = \tau \omega = |M \times B| \omega = (MB \sin \theta) \omega \]

\[ = \frac{\omega A^2B \sin \omega t}{R} B \sin \omega t \omega = P_{elec} \]
Example: A coil of wire containing 500 circular loops with radius 4.00 cm is placed in a uniform magnetic field. The direction of the field is at an angle of 60° with respect to the plane of the coil. The field is decreasing at a rate of .200 T/s. What is the magnitude of the induced emf?
A determination of B vs r (lab experiment)

\[ I = I_0 \sin(\omega t) \]

\[ B(r, t) = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I_0}{2\pi r} \sin(\omega t) \]

\[ \Phi \approx B(r, t) A_{\text{coil}} \]

\[ E_{\text{coil}} = -\omega NA \frac{\mu_0 I_0}{2\pi r} \cos(\omega t) \]

\[ E_{\text{coil}} \propto \frac{1}{r} \]
Eddy Currents

Induced currents in bulk conductors
magnetic forces on induced currents
energy losses to resistance

ex: Conducting disk rotating through a perpendicular magnetic field

Ferromagnets ~ Iron (Electrical Conductor)
Constrain Eddy currents with insulating laminations
Induced Electric Fields

Changing magnetic Flux produces an EMF $\mathcal{E}$

Induced Electric Field

$$\mathcal{E} = \oint E \cdot d\vec{l}$$

$$\oint E \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Not an Electrostatic field

Not Conservative!
\[ \oint E \cdot d\vec{l} = 2\pi r E \]
\[ = \left| \frac{d\Phi_B}{dt} \right| \]
\[ E = \frac{1}{2\pi r} \left| \frac{d\Phi_B}{dt} \right| \]
Maxwell’s Equations

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o} \]

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]

\[ \oint \vec{B} \cdot d\vec{l} = \mu_o (I_c + \varepsilon_o \frac{d\Phi_E}{dt}) \]

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_M}{dt} \]

\[ \nabla \cdot \vec{E} = \frac{4\pi \rho}{\varepsilon_o} \]

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{B} = \mu_o (\vec{J}_c + \varepsilon_o \frac{d\vec{E}}{dt}) \]

\[ \nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \]

Lorentz Force Law

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]
Superconductivity

zero resistivity below $T_c$

(with no external magnetic field)

with field above the critical field $B_c$, no superconductivity at any temperature
More magnetic fun with superconductivity: the Meissner Effect

Within a superconducting material, $B_{\text{inside}} = 0$!

magnetic field lines are expelled from superconductor

Relative permeability $K_m = 0$

internal field is reduced to 0

$\Rightarrow$ Superconductor is the perfect diamagnetic material!
mechanical effects

ferromagnetic or paramagnetic materials attracted to permanent magnet

superconductor repels permanent magnet!

Type-II superconductors

small filaments of “normal phase” coexist within bulk superconductor

some magnetic field lines penetrate material

two critical fields field just begins to penetrate material

$B_{c1}$ : magnetic field just begins to penetrate material

$B_{c2}$ : bulk material ”goes normal”

more practical for electromagnets: higher $T_c$ and $B_{c2}$