Chapter 23: Electric Potential

Electric Potential energy

Work done by a force acting on an object moving along a path:

\[
W_{a \to b} = \int_{a}^{b} \vec{F} \cdot d\vec{l}
\]

If the force is a conservative force, the work done can be expressed as a change in potential energy:

\[
W_{a \to b} = U_a - U_b
\]

Work-Energy Theorem: the change in kinetic energy equals the total work done on the particle.

If only conservative forces are present:

\[
K_b - K_a = W_{a \to b} = U_a - U_b \Rightarrow K_a + U_a = K_b + U_b
\]
A charge in a uniform electric field

\[ \vec{F} \cdot d\vec{l} = q\vec{E} \cdot d\vec{l} = qE(-dy) \]

\[ W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{y_{a}}^{y_{b}} qE(-dy) = qE(y_{a} - y_{b}) \]

\[ U = qEy \]
The work done on a charge $q'$ in the presence of another charge $q$

$$\vec{F} \cdot d\vec{l} = k \frac{q'q}{r^2} dr$$

$$W_{a\to b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{r_{a}}^{r_{b}} k \frac{q'q}{r^2} dr$$

$$= -k \frac{q'q}{r} \bigg|_{r_{a}}^{r_{b}} = kq'q \left( \frac{1}{r_{a}} - \frac{1}{r_{b}} \right)$$

$$= U_{a} - U_{b}$$

$$U(r) = k \frac{q'q}{r} \text{ convention } U(r = \infty) = 0$$

for several source charges

$$U_{q'} = kq' \left( \frac{q_{1}}{r_{1}} + \frac{q_{2}}{r_{2}} + \frac{q_{3}}{r_{3}} + \cdots \right) = kq' \sum_{i} \frac{q_{i}}{r_{i}}$$

$$U_{total} = \sum_{\text{all pairs}} k \frac{q_{i}q_{j}}{r_{ij}}$$
Potential

Potential is potential energy per unit charge

\[ V = \frac{U}{q'} \quad \text{or} \quad U = q' V \]

units : \(1V = 1\text{Volt} = 1J / C\)

Potential is often referred to as “voltage”

In terms of “work per charge”

\[ \frac{W_{a\rightarrow b}}{q'} = \frac{U_a - U_b}{q'} = V_a - V_b \]

In terms of source charges

\[ V = k \sum \frac{q_i}{r_i} \quad \text{or} \quad V = k \int \frac{dq}{r} \]
Potential and potential differences from the Electric Field

\[ W_{a \rightarrow b} = \int_{a}^{b} q' \vec{E} \cdot d\vec{l} \]

\[ V_a - V_b = \int_{a}^{b} \vec{E} \cdot d\vec{l} \]

\[ V_{ab} = V_a - V_b = -\int_{b}^{a} \vec{E} \cdot d\vec{l} \]

\[ dV = - \vec{E} \cdot d\vec{l} \sim F_x = - \frac{dU}{dx} \]
Electric Potential (continued): Examples

Electron Gun: What is the speed of an electron accelerated from rest across a potential difference of 100V? What is the speed of a proton accelerated under the same conditions? (http://phys23p.sl.psu.edu/phys_anim/EM/egun.avi)

An electric dipole oriented vertically at the origin consists of two point charges, +/- 12.0 nC placed 10 cm apart. What is the potential at a point located 12 cm from the dipole in the horizontal direction? What is the potential energy associated with a +4.0 nC charge placed at this point?
Spherical Charged Conductor

Outside: looks like a point charge

Inside: field is zero \[ dV = -\vec{E} \cdot d\vec{l} = 0 \]

\[
E = k \frac{q}{R^2}
\]

at the surface of the sphere:

\[
V = k \frac{q}{R}
\]

\[ V_{\text{max}} = E_{\text{max}} R \implies \text{Larger E with smaller R} \]

Dielectric Strength = maximum electric field strength an insulator can withstand before Dielectric Breakdown (Insulator becomes a conductor).

\[ \implies \text{High voltage terminals have large radii of curvature} \]

\[ \implies \text{“Sharp” surfaces better able to produce sparks} \]

“lightning” demo
Potential between parallel plates

Uniform field: \( U = qEy \)

\[ => V = Ey = (V_y - V_b) \]

\[ V_{ab} = V_a - V_b = Ed \]

or

\[ E = \frac{V_{ab}}{d} \]

(True for uniform fields only, although this can provide an estimate of field strength)
Line charge and (long) conducting cylinder

\[ E_r = \frac{2k\lambda}{r} \]

\[ V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} \]

\[ = \int_{r_a}^{r_b} \frac{2k\lambda}{r} dr = 2k\lambda \ln \frac{r_b}{r_a} \]

\[ = 2k\lambda \left( \ln r_a - \ln r_b \right) \]

\[ = 2k\lambda \ln \frac{r_a}{r_b} \]
Equipotential surfaces physlet: http://phys23p.sl.psu.edu/simulations/physlets/em_fieldplot.html

A surface in space on which the potential is the same for every point.

Surfaces of constant voltage.
Potential Gradient

\[ V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = -\int_b^a \vec{E} \cdot d\vec{l} \]

\[ V_a - V_b = \int_b^a dV = -\int_b^a \vec{E} \cdot d\vec{l} \Rightarrow dV = -\vec{E} \cdot d\vec{l} \]

\[ dV = E_l dl \]

\[ E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \]

\[ \vec{E} = -\left( \hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \right) = -\left( \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right)V \]

\[ \vec{E} = -\nabla V \]
Millikan oil-drop experiment
Experimental investigation into the quantization of charge

small drops of oil, with small amounts of excess charge.

naive: balance $\rightarrow q = mg/E$

need mass! (or drop radius + density)

Balance gravity, electric force and air resistance on drop in motion:

$F_f = 6\pi \eta r \nu$

$F'_f = 6\pi \eta r \nu'$

$F_g \, mg = \frac{4}{3} \pi r^3 \rho g$

$F_E = qE$
Millikan’s (and later) results

thousands of measurements => all (positive or negative) integer multiples of $e$!

=> charge is quantized!!

Modern Quantum Chromodynamics:

quarks have fractional ($\pm 1/3e \enspace \pm 2/3e$) charges, but never appear alone (“confinement”) net charge of all observed objects are integer multiples of $e$. 
Cathode Ray tube

\[
\text{electron gun } v_x = \sqrt{\frac{2eV_1}{m_e}}
\]

between plates \( a_y = \frac{eE}{m_e} = \frac{eV_2}{m_e d} \)

\( v_y = a_y t \quad L = v_x t \)

\( v_y = \frac{eV_2}{m_e d} \frac{L}{v_x} \)

\( \tan \theta = \frac{v_y}{v_x} = \frac{eV_2}{m_e d} \frac{L}{v_x^2} = \frac{eV_2}{m_e d} \frac{L m_e}{2 e V_1} = \frac{V_2 L}{d 2 e V} \)