Chapter 25: Current, Resistance and Electromotive Force

Charge carrier motion in a conductor in two parts

Constant Acceleration

\[ \vec{F} = m\ddot{a} = q\vec{E} \]

Randomizing Collisions (momentum, energy)

=> Resulting Motion  http://phys23p.sl.psu.edu/phys_anim/EM/random_walk.avi

Average motion = Drift Velocity = \( v_d \sim 10^{-4} \text{ m/s} \)

Typical speeds \( \sim 10^6 \text{ m/s} \)
Current Flow = net motion of charges
= Charge carriers
    charge q
    speed $v_d$

$I = \text{Current} = \text{rate at which charge passes the area}$

$$I \equiv \frac{dQ}{dt} \quad \text{units:} \quad \frac{1C}{1S} = 1 \text{Amp} = 1 \text{A}$$

Negative charge carriers move in opposite direction of conventional current.
Connection with microscopic picture:

\[
\begin{align*}
E & \quad A \\
q & \quad v_d \\
\ell &= v_d \, dt
\end{align*}
\]

\[dQ = \text{charge that passes through } A\]
\[= \text{number that pass through } A \times \text{charge on each}\]
\[= (n \, A \, v_d \, dt) \, q, \quad n = \text{number density} = \text{number/volume}\]

\[I = \frac{dQ}{dt} = nq v_d \, A\]

Current Density:

\[J \equiv \frac{I}{A} = nq v_d\]

\[\vec{J} = nq \vec{v_d}\]

(work for negative charge carriers, multiple types of charge carriers as well)

\[\vec{J} = n_1 q_1 \vec{v}_{d1} + n_2 q_2 \vec{v}_{d2} + \ldots\]
Example: 18 gauge copper wire (~1.02 mm in diameter)  
- constant current of 2A  
- \( n = 8.5 \times 10^{28} \ m^{-3} \)  (property of copper)  
find \( J, \nu_d \)
Current as a response to an applied electric field

$$\vec{J} = \vec{J}(\vec{E})$$

$$= \sigma \vec{E}$$

$$\sigma = \text{conductivity}$$

$$\rho \equiv \frac{E}{J} = \frac{1}{\sigma} = \text{resistivity}$$

units = \frac{V/m}{A/m^2} = \frac{V}{A} m = \Omega m$$
\[ \rho \text{ depends upon} \]
- material
- \( E \)
- Temperature

If \( \rho \) does not depend on \( E \), the material is said to obey
“Ohm’s Law”
For a cylindrical conductor

\[ E = \rho J \]

\[ V_{ab} = V = E\ell \]

\[ = \rho J\ell = I\frac{\rho \ell}{A} \]

\[ R = \frac{V}{I} = \frac{\rho \ell}{A} = \text{resistance} \]

\[ V = IR \]

*see also example 28-3 re: alternate geometries*

Example: 50 meter length of 18 gauge copper wire (~1.02 mm in diameter)
constant current of 2A
\[ \rho = 1.72 \times 10^{-8} \, \Omega \cdot m \]
find \( E, V, R \)
Temperature Dependence of $\rho$

For small changes in temperature:

$$\rho_T = \rho_o [1 + \alpha (T - T_o)]$$

$\alpha$ = temperature coefficient of resistivity

$$R_T = R_o [1 + \alpha (T - T_o)]$$
## Resistor Color Codes

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
<th>Color</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>black</td>
<td>0</td>
<td>none</td>
<td>±20%</td>
</tr>
<tr>
<td>brown</td>
<td>1</td>
<td>silver</td>
<td>±10%</td>
</tr>
<tr>
<td>red</td>
<td>2</td>
<td>gold</td>
<td>±5%</td>
</tr>
<tr>
<td>orange</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>yellow</td>
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<tr>
<td>green</td>
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<tr>
<td>blue</td>
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<tr>
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<tr>
<td>gray</td>
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<td></td>
</tr>
<tr>
<td>white</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Value: \( n_1 n_2 \times 10^{n_3} \pm x\% \)

10x10^2\pm5\%
Electromotive Force and Circuits

Steady current
  requires a complete circuit
  path cannot be *only* resistance
  cannot be only potential drops in direction of current flow

Electromotive Force (EMF)
  provides *increase* in potential $\mathcal{E}$
  converts some external form of energy into electrical energy

Single emf and a single resistor:

\[
V = IR = \mathcal{E}
\]
Measurements

Voltmeters measure Potential Difference (or voltage) across a device by being placed in parallel with the device.

Ammeters measure current through a device by being placed in series with the device.
Real Sources and Internal Resistance

Ideal emf $\mathcal{E}$
- determined by how energy is converted into electrical energy

Internal Resistance $r$
- unavoidable “internal” losses
- aging batteries $\Rightarrow$ increasing internal resistance
Open Circuit
\[ I = 0 \]
\[ V_r = 0 \]
\[ V_{ab} = \mathcal{E} \]

Short Circuit
\[ V_r = Ir = \mathcal{E} \]
\[ V_{ab} = 0 \]
Complete Circuit

\[ \mathcal{E} = Ir + IR \]

\[ I = \frac{\mathcal{E}}{R + r} \]

\[ V_{ab} = \mathcal{E} - Ir \]

Charging Battery

\[ V_{ab} = \mathcal{E} + Ir \]
Energy and Power

\[ dW = dQ V_{ab} = V_{ab} I dt \]

\[ P = \frac{dW}{dt} = IV_{ab} \]

\[ V = IR \]

Resistance

\[ P = IV = I^2 R = \frac{V^2}{R} \]

Ideal emf

\[ V = \mathcal{E} \]

Power delivered by (I out of +) or to (I into of +) battery
Power and Real Sources

**Discharging Battery**

\[ V_{ab} = \mathcal{E} - Ir \]

Power delivered externally: \[ IV_{ab} = I\mathcal{E} - I^2r \]

**Charging Battery**

\[ V_{ab} = \mathcal{E} + Ir \]

Power delivered by external source: \[ IV_{ab} = I\mathcal{E} + I^2r \]
Real Battery with Load

\[
I = \frac{\mathcal{E}}{R + r}
\]

\[
V_R = IR = \frac{\mathcal{E} R}{R + r} = \frac{\mathcal{E}}{1 + r/R}
\]

\[
P_\mathcal{E} = IE = \frac{\mathcal{E}^2}{R + r}
\]

\[
P_r = I^2 r = \left(\frac{\mathcal{E}}{R + r}\right)^2 r
\]

\[
P_R = I^2 R = \left(\frac{\mathcal{E}}{R + r}\right)^2 R
\]

\[
eff = \frac{P_R}{P_\mathcal{E}} = \frac{1}{1 + r/R}
\]

\[
\max P_R
\]

\[
\frac{dP_R}{dR} = 0 \Rightarrow R = r
\]

"Impedance Matching"
Complete Circuit Example

\[ I = \]
\[ V_{ab} = \]
\[ P_\varepsilon = \]
\[ P_r = \]
\[ P_R = \]
\[ P_{V_{ab}} = \]
Theory of Metallic Conduction

Constant Acceleration between randomizing collisions (momentum, velocity randomized)

\[ \rho = \frac{E}{J} \]

\[ \vec{J} = nq\vec{v}_d \]

\[ \vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} \]

\[ \vec{v} = \vec{v}_o + \vec{a}\tau \]

\[ \vec{v}_{avg} = \vec{a}\tau = \frac{q\vec{E}}{m}\tau = \vec{v}_d \]

\[ \vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m} \vec{E} \]

\[ \rho = \frac{m}{nq^2\tau} \]

\[ \tau = \text{mean time between collisions} \]

\[ \ell = \text{mean free path} \]

\[ = (\vec{v}_o)_{avg} \cdot \tau \]
Example: What is the mean time between collisions and the mean free path for conduction electrons in copper?

\[ \rho = 1.27 \times 10^{-8} \text{Ω} \cdot \text{m} \]

\[ n = 8.5 \times 10^{28} \text{m}^{-3} \]

\[ m = 9.1 \times 10^{-31} \text{kg} \]

\[ q = e = 1.6 \times 10^{-19} \text{C} \]

\[ \nu_o \approx 1 \times 10^6 \frac{\text{m}}{\text{s}} \]
Physiological Effects of Current

Nerve action involves electrical pulses
currents can interfere with nervous system

~.1A can interfere with essential functions
(heartbeat, e.g.)
currents can cause involuntary convulsive muscle action

~.01 A

Joule Heating ($I^2R$)

With skin resistance

dry skin: $R \sim 500k\Omega$
wet skin: $R \sim 1000\Omega$