8: Potential Energy and Conservative Forces

Conservative and Nonconservative Forces

Example: the force of gravity vs. friction

Work done to raise an object a height $h$: $W = mgh$

- Work done by gravity on object if the object descends a height $h$.

The work done lifting the object can be recovered (released) at some later time!

In contrast with the work done by/against friction

this work cannot be recovered

Consider the net work moving an object around a closed path:

- net work done by gravity is zero (moving up, over, down, back)
- net work done by friction is NOT zero (moving left, forward, right, back)

Definition: A **conservative force** does zero work on *any* closed path

break the closed path into a trip from $a$ to $b$ and then back(along alternate routes):

$W_2 = -W_1$
$W_3 = -W_1$

so $W_2 = W_3$

For a conservative force, the work done is independent of the path taken, and depends only on the endpoints
Potential Energy

energy associated with position
example: gravitational potential energy

Work done to raise an object a height $h$: $W = mgh$

= Work done by gravity on object if the object descends a height $h$.

identify source of work as Potential Energy

$U = mgy$

*Work done by a conservative force is the loss of potential energy*

$W_c = -\Delta U$

note: $U$ depends upon reference for $y=0$, but only change in potential is physically important

Example: Find the gravitational potential energy of a 65 kg person on a 3m high diving board, taking $U=0$ at the water level.
Example: An 82.0 kg climber is in the final stage of the ascent of 4301-m high Pike's Peak. What is the change in gravitational potential energy as the climber gains the last 100.0m of altitude? Take \( U = 0 \) at sea level and repeat by taking \( U = 0 \) at the top of the peak.
Example: A candy bar has a calorie content of 210.0 Cal = 210.0 kcal. Each kcal is equivalent to 4186 J. If an 82.0 kg mountain climber eats the bar and magically converts it all to potential energy, what would be his gain in altitude?
Elastic Potential Energy

energy stored in stretching or compressing a spring

Work done compressing: \( W = \frac{1}{2} k x^2 \)

= work that can be extracted by releasing the spring

\( U = \frac{1}{2} k x^2 \)

Example: When a force of 120.0 N is applied to a certain spring, it causes a stretch of 2.25 cm. What is the potential energy of this spring when it is compressed by 3.50 cm?
Conservation of Mechanical Energy

Conservation Principle: For an isolated system, a conserved quantity keeps the same value no matter what changes the system undergoes.

Consider a system acted upon \textbf{only} by conservative forces: \[ W_c = \Delta K \]

In terms of potential energy: \[ W_c = -\Delta U \quad \text{so} \quad \Delta K = -\Delta U \]

Conservation of Mechanical Energy: The total amount of energy in a system acted upon only by conservative forces always remains constant:

\[ E \equiv K + U \]

\[ E_i = K_i + U_i = K_f + U_f = E_f \]

energy transformations from one form to another may (will!) occur
Example: Suppose a student throws his 0.120 kg graduation cap straight up with a speed of 7.85 m/s. Use conservation of energy to determine the maximum height the cap reaches.

*(this is a little different than text example)*
Example: A 1.70 kg block slides on a horizontal frictionless surface until it encounters a horizontal spring with a force constant of 955 N/m. The block comes (momentarily) to rest when the spring is compressed by 4.60 cm. What is the initial speed of the block before it collides with the spring?
Nonconservative forces

\[ W_c + W_{nc} = \Delta K \]

so \[ W_{nc} = \Delta E = \Delta K + \Delta U \]

eg: Friction generally results in *loss* of energy

\[ E_{\text{initial}} = E_{\text{final}} + |W_{\text{friction}}| \]

Example: A 17 g leaf falls from a tree in a forest. It drops straight down from a height of 5.3 m above the ground, and its speed upon landing on the ground is 1.3 m/s. How much work was done by nonconservative force?
Example: A mass $m_1 = 2.40$ kg is connected to a second mass $m_2 = 1.80$ kg, as shown. When the blocks are released from rest, they move through a distance $d=0.500$ m, at which point $m_2$ hits the floor. Given that the coefficient of kinetic friction between $m_1$ and the board is $\mu_k = 0.450$, determine the final speed of the blocks.
Potential Energy Curves

\[ E_0 = K + U = E \]

\[ K + U = E \]

Turning points

Stable vs. Unstable (vs neutral) Equilibrium

Oscillations about Equilibrium \( \frac{1}{2} kx^2 \): looks parabolic

In 2-D

Equipotentials: like equal altitude contours on a contour map