Momentum: a measure of motion
Force: a cause of change in motion
What changes when a force is applied?
\[ \vec{F} = m \vec{a} = m \frac{\Delta \vec{v}}{\Delta t} \]; but mass does not change
\[ \vec{F} = \frac{\Delta m \vec{v}}{\Delta t} \]; \( m \vec{v} \) changes with time

\text{Linear Momentum: } \vec{p} \equiv m \vec{v} \quad (\text{vector!})
the tendency of an object to pursue straight line motion

Example: What is the magnitude of the momentum of a 1180 kg car traveling at 30.0 mph (13.4 m/s)?
What is the magnitude of the momentum of a .142 kg baseball thrown with a speed of 101 mpg (45.1 m/s)?
Impulse: the change in motion

\[ J = \Delta \vec{p} = m\Delta \vec{v} \]

since \( \Delta \vec{v} = \vec{a}\Delta t \)

and \( \vec{F} = m\vec{a} \)

\[ J = \vec{F}\Delta t = \Delta \vec{p} = \Delta m\vec{v} \]

Example: A .144 kg baseball moving towards home plate with a speed of 43.0 m/s is bunted by a bat which exerts an average force of 6.50E3 N on the ball in the direction of the pitcher for 1.30 ms. What is the impulse on the ball and what is its final speed?
Conservation of momentum

two (or more) bodies + action/reaction + no *external* forces

\[ \mathbf{F}_{AB} = - \mathbf{F}_{BA} \]

→ equal but opposite impulses

→ \( \Delta \mathbf{p}_A + \Delta \mathbf{p}_B = 0 \)

*When the net external force on a system is zero, the total momentum of that system is constant.*

\[ \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \cdots \] is constant

Collisions, explosions etc: \( m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_1 \mathbf{v}'_2 \)
Example: Two canoes meet in the middle of a lake, whereupon a person in one canoe pushes on the other canoe with a force of 46 N to separate the canoes. If the first canoe and occupants have a total mass of 30 kg and the second canoe and occupants have a mass of 250 kg, determine the momentum and speed of each canoe after 1.20 s of pushing. (What is the TOTAL initial and final momentum of the two canoe system?)
Collisions (in the absence of external forces)

**Elastic Collisions**
- conserve KE (total KE is the same before and after collision)
- this is a “springy” collision

**Inelastic Collisions**
- some KE is lost during collision (heat, sound, etc.)
- this is a somewhat “sticky” collision

**Completely Inelastic Collisions**
- objects stick together
- maximum possible loss of KE

In all collisions, the total momentum is conserved!
Example: A 1200 kg car moving at 2.5 m/s is struck in the rear by a 2600 kg truck moving at 6.2 m/s. If the vehicles stick together after the collision, what is the speed of the wreckage just after the collision?
Example: A 95.0 kg running back moving at 3.75 m/s collides head on with a 111 kg linebacker moving in the opposite direction at 4.10 m/s who makes a successful tackle. What is the speed of the players just after the collision? What is their total initial and final kinetic energies?
Chapter 8: Applying Conservation Laws

Elastic collisions

conservation of KE + conservation of momentum
in one dimension. impactor \((m_1)\) hits target \((m_2)\) at rest

\[
m_1 v_0 = m_1 v_{1,f} + m_2 v_{2,f}
\]

\[
\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2
\]

+ algebra:

\[
v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_0
\]

\[
v_{2,f} = \frac{2m_1}{m_1 + m_2} v_1
\]

*track demo's!!!!!!*
Example: A 96 kg bumper car moving 1.24 m/s bounces (elastically) off a stationary 135 kg bumper car at rest. What are the final velocities of the cars?
Center of Mass
aka Center of Inertia
“average” location of mass of a system of particles

\[ x_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m_i x_i}{\sum m_i} \text{ (similarly } y_{cm}, z_{cm}) \]

\[ \bar{r}_{cm} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m_i \bar{r}_i}{\sum m_i} \]

the center of mass of a system is also the balance point where the system can be balanced (in a uniform gravitational field)
→ aka the center of gravity
the center of mass of an object does NOT need to be located in the physical body.
horseshoe, eg

External forces and the motion of center of mass

\[ M\ddot{\bar{r}}_{cm} = m_1\ddot{\bar{r}}_1 + m_2\ddot{\bar{r}}_2 + \cdots = \sum \ddot{F} = \sum \ddot{F}_{ext} + \sum \ddot{F}_{int} \]

action – reaction \( \sum \ddot{F}_{int} = 0 \)

\[ M\ddot{\bar{r}}_{cm} = \sum \ddot{F}_{ext} = \frac{d\vec{P}}{dt} \]

since \( M\ddot{\bar{r}}_{cm} = M \frac{d^2\bar{v}_{cm}}{dt^2} = \frac{d(M\bar{v}_{cm})}{dt} \)
Example: Suppose two masses are separated by a distance of .500 m. One of the masses is .260 kg and the other is .170 kg. How far from the first mass is the center of mass of this system?