12: Gravity

Newton's Law of Universal Gravitation

Interaction between masses

responsible for things falling to earth as well as planetary orbits

Unifying description of heavenly and earthly phenomena!

\[ F = G \frac{m_1 m_2}{r^2} \]

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

example: What is the gravitational attraction between a (110 kg) man and his (11.2 kg) dog when they are separated by 1.00 m and when they are separated by 10.0 m?
Spherical bodies act as point masses with the mass concentrated at the center of the body. An object of mass $m$ is separated from the center of the earth by a distance equal to the radius of the earth, $R_E$, so

$$F = G \frac{mM_E}{R_E^2}$$

$$F = mg, \text{ so}$$

$$g = G \frac{M_E}{R_E^2}$$

Example: If $g$ is 9.81 m/s$^2$ at sea level, what is it 5.50 miles above sea level?
example: What is the acceleration of gravity on the surface of the moon?

given \( G \) and that the acceleration of gravity is 9.81 \( \text{m/s}^2 \), what is the mass of the Earth? What is the Earth's average density?
Kepler's observations of planetary orbits (aka *Kepler's Laws*)

The orbits of the planets about the sun are ellipses, with the sun at one focus.

From Newton's mechanics, there are only two types of forces which can give closed ellipses as orbits:

- Force proportional to distance (like a spring)
- Force inversely proportional to distance squared (Newton's Gravity!)

As a planet moves along its orbit, it sweeps out an equal area in an equal time:

follows from conservation of angular momentum \( mvr = mv'r' \)

The square of the period of a planet's orbit is proportional to the cube of the orbit's semi major axis:

follows from \( F = ma \)

Consider circular orbits:

\[
F = G \frac{M_S m}{r^2}
\]

\[
F = ma_{cp} = m \frac{v^2}{r}
\]

with \( v = \frac{2\pi r}{T} \)

\[
G \frac{M_S m}{r^2} = m \frac{4\pi^2 r^2}{r T^2}
\]

\[
T^2 = \frac{4\pi^2}{GM_S} r^3
\]
example: The Earth orbits the sun once a year at an average distance of 1.5E11 m. Use this information to determine the mass of the sun. Determine the period of Mercury's orbit given that it orbits the sun at a distance of 5.79E10M.
example: What is the *altitude* of a geosynchronous satellite?
Gravitational Potential Energy (via calculus)
potential taken to be zero at infinite separation of the masses
\[ U = -G \frac{M}{r^2}m \]

*with energy conservation*
\[ E = \frac{1}{2} m v^2 - G \frac{M}{r^2}m \]

*dropping an object & very far away*
\[ E_i = 0 = E_f = \frac{1}{2} m v_f^2 - G \frac{M}{r_f^2}m \]
\[ v_f = \sqrt{\frac{2GM}{R}} \]
Example: How fast would an asteroid be moving when it hit the earth, assuming it started very far away at low speeds.

Example: What is the escape speed from the surface of the earth?
Black holes: escape speed is greater than the speed of light! Schwarzschild radius is distance of no return! What is the Schwarzschild of a one solar mass black hole?