This poster presents the underlying physics used to simulate motions typically discussed under the auspices of ‘Roller Coaster Physics’ in animation and an interactive Java applet used in introductory physics. The car is modeled as a mass moving along a parametric curve, either at constant speed or coasting (with and without friction). Generating solutions to the resulting equations of motion places some constraints on the parametric equations describing the path of the track through space. The resulting materials are used to explore topics such as energy, power, circular motion, reaction forces and friction. In order to dramatize the repercussions of failing to loop at sufficient speed, collision dynamics are employed to simulate the car bounding off the track and ground. Visual materials generated using the model are available online.\(^{(1,2)}\)

### Dynamics 1: Constant Speed

**Parameterized Path:** \( \vec{r}(s) \)
- Continuous
- 2\(^{nd}\) order differentiable

**Define:** \( \dot{\vec{r}} = \frac{d\vec{r}}{dt} \) and \( \ddot{\vec{r}} = \frac{d\dot{\vec{r}}}{dt} \) then \( \ddot{\vec{r}} = \ddot{\vec{r}}\hat{s} \)

**Speed** \( v = |\dot{\vec{r}}| = |\dot{\vec{r}}|\hat{s} \)

**Numerically integrate** \( ds = \frac{v}{|\dot{\vec{r}}|}dt \) to get \( s(t), \vec{r}(s(t)) \)

**Some useful derivative fun:**
\[
\dot{s} = \frac{v}{|\dot{\vec{r}}|} = v(\vec{r}' \cdot \vec{r}'')^{-1/2} \\
\ddot{s} = \dot{v}(\vec{r}' \cdot \vec{r}'')^{-1/2} - v^2(\vec{r}' \cdot \vec{r}'')^{-2} \vec{r}' \cdot \vec{r}'' - \vec{r}' \\
\dddot{s} = \ddot{v}(\vec{r}' \cdot \vec{r}'')^{-1/2} - v^2 \dddot{\vec{r}}' \cdot \vec{r}'' - \vec{r}' \\
\dot{\vec{r}} = \vec{r}' = \frac{v}{|\dot{\vec{r}}|} \left( \frac{\dot{\vec{r}}}{\|\dot{\vec{r}}\|} - v^2 \frac{\vec{r}' \cdot \vec{r}''}{\|\dot{\vec{r}}\|^2} \right)
\]

**Reaction Force:** \( \vec{F} \quad m\ddot{\vec{r}} \)

**Perceived “g-force”:** \( \frac{\vec{F}}{mg} \)

**Dynamics 2.0: Coasting**

**Application of Lagrangian Dynamics (3):** \( L = T - V \)
\[
L = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{r}' \cdot \vec{r}'')s^2 \\
V = mg\dot{\vec{r}} \cdot \vec{r}
\]

**From Lagrange’s Equation**
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{r}}} - \frac{\partial L}{\partial \vec{r}} = 0 \\
m(\vec{r}' \cdot \vec{r}'')\dot{s}^2 + mgy' = 0 \\
\dot{s} = -\frac{(v^2\dddot{s})\dot{s} + gy'}{v^2\dddot{s}}
\]

**Integrate:**
\[
s(t + dt) = \bar{s}(t) + \dot{s}(t)dt \\
\dot{s}(t + dt) = \dot{s}(t) + \ddot{s}(t)dt
\]

**Dynamics 2.1: Coasting with Friction**

**More Lagrangian Dynamics!**

Rayleigh’s dissipation function \( \Phi = \frac{1}{2}bv^2 \)

**Modified Lagrange’s Equation**
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{r}}} - \frac{\partial L}{\partial \vec{r}} + \frac{\partial \Phi}{\partial \dot{\vec{r}}} = F \\
m(\vec{r}' \cdot \vec{r}'')\dot{s}^2 + mg\dot{\vec{r}}' \cdot \vec{r}'' + b\dot{\vec{r}}' \cdot \vec{r}'' = 0
\]

**Fine-Tune Energy conservation:**
\[
dW_f = \dot{\vec{K}} \cdot d\vec{r} = -b\dddot{\vec{r}} \cdot d\vec{r} = -b(\vec{r}' \cdot \vec{r}'')s^2 \\
E = \vec{K} + \Phi
\]

**Adjust speed each iteration to scale KE to “fix” Energy**

**Dynamics 3: Bouncing**

**Consequences of “Failure to Loop”**

Freefall between “bounces” from track

Collision Dynamics: Impulse approximation for interacting bodies a la Video Game Physics.\(^{(4)}\)

- Conservation of momentum, angular momentum
- Coefficient of restitution \( c \)
- “Massive” body 1 \( m_1^{-1} = 0, \quad l_1^{-1} = 0 \) for track

**Impulse:**
\[
\vec{F} = \frac{m_2v_2}{m_1} \frac{1}{l_2} \quad \Delta \vec{\omega}_2 = \Delta \vec{r}_2 \times \vec{F} \\
\Delta \vec{v}_2 = \frac{\vec{F}}{m_2} \quad \Delta \vec{r}_2 = \vec{F} \times \vec{r}_2
\]

**Implemented in 2-D**

**Interactive explorations of topics in energy conservation, reaction forces, friction and g-forces.**

**Animation:** Frames created with POV-Ray 3-D ray tracing program.\(^{(6)}\)

**Coasting and Failing to Coast examples**

**First person views and non circular loop**

With additional physical data (radius, mass, speed, initial height, etc) students tasked with determining:
- Minimum speed to power through loop at constant speed
- Maximum power required during successful loop at constant speed
- G-forces, reaction forces
- Minimum height to coast through circular loop

Some of this project is from Animations for Physics and Astronomy at Penn State Schuylkill. \(^{(7)}\)

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1. Roller Coaster Model (Java applet)
2. Roller Coaster Physics Animation http://www.youtube.com/watch?v=byD29Ht8SU