Beyond Hooke's Law in Scale Bungee Jumping with Rubber Bands

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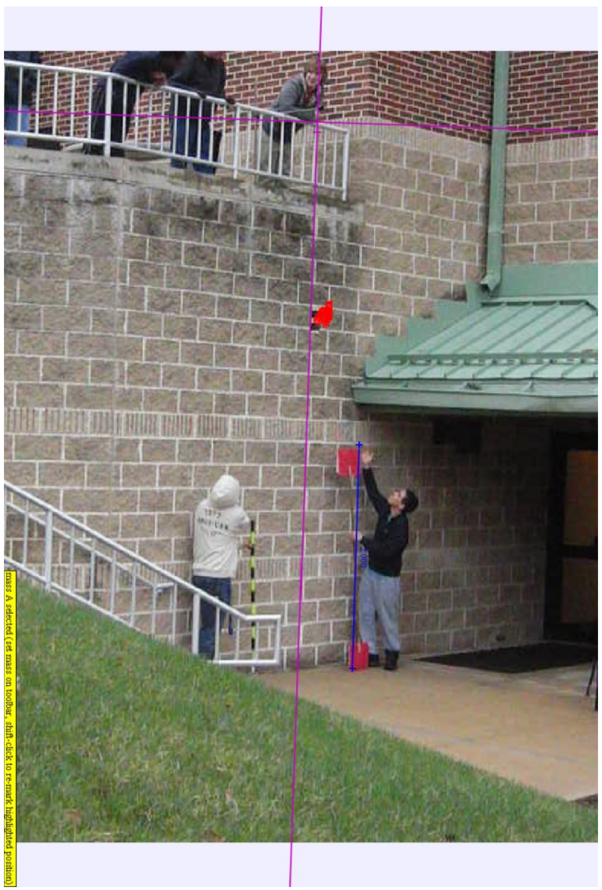
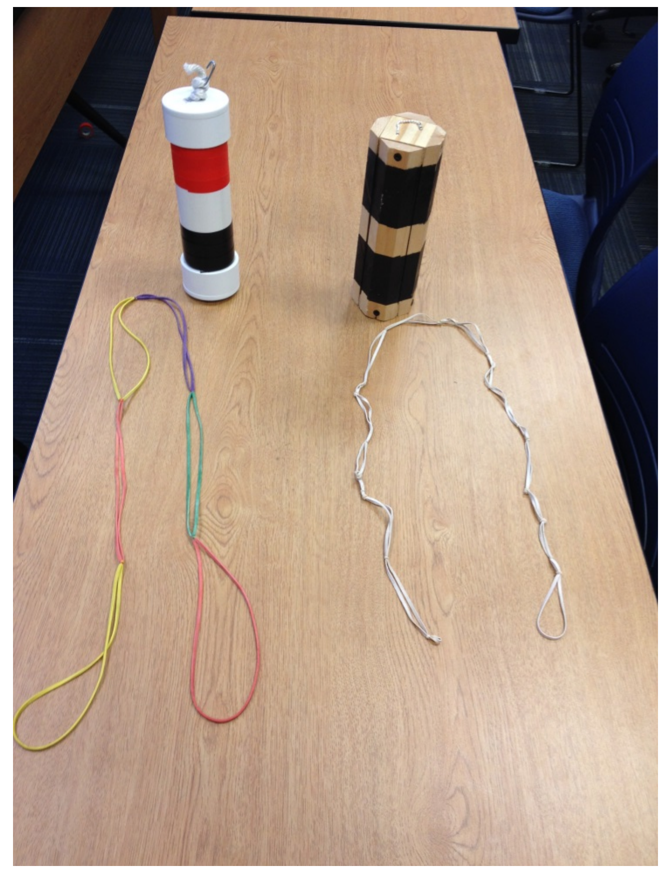
Scaled bungee jumping activities are popular at many levels, often implemented as "Barbie Bungee Jumping" using dolls or action figures and elastic cord made from rubber bands. The departure from Hooke's law in the elastic behavior of the cord can be significant in these activities, and can be a worthwhile part of a discussion of Hooke's law and its limitations. In this presentation, the measurement of features such as non-linearity, hysteresis, and plastic deformation measured as part of a pre-drop activity will be discussed. An Easy Java-Script Simulation of a bungee jumper has been developed which incorporates a student measured model of the hysteresis envelope and a phenomenological model of the effects of hysteresis during the dynamics of the drop. This simulation will be presented. Finally, some techniques that facilitate the extraction of data from video analysis programs like Tracker will be discussed.

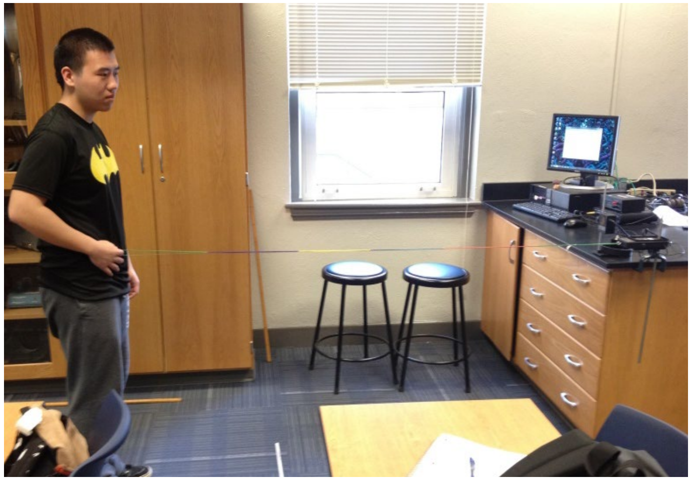
Beyond Hooke's Law in Scale Bungee Jumping with Rubber Bands

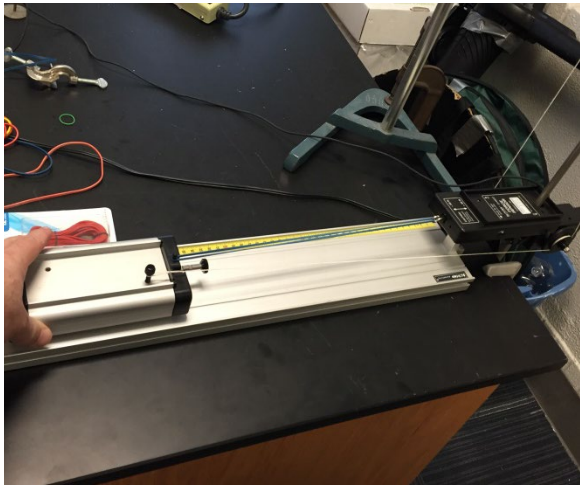
(AKA more physics fun in Barbie Bungee Jumping)





Cord Properties Measurement





Full Size Cords (Chains of Bands)

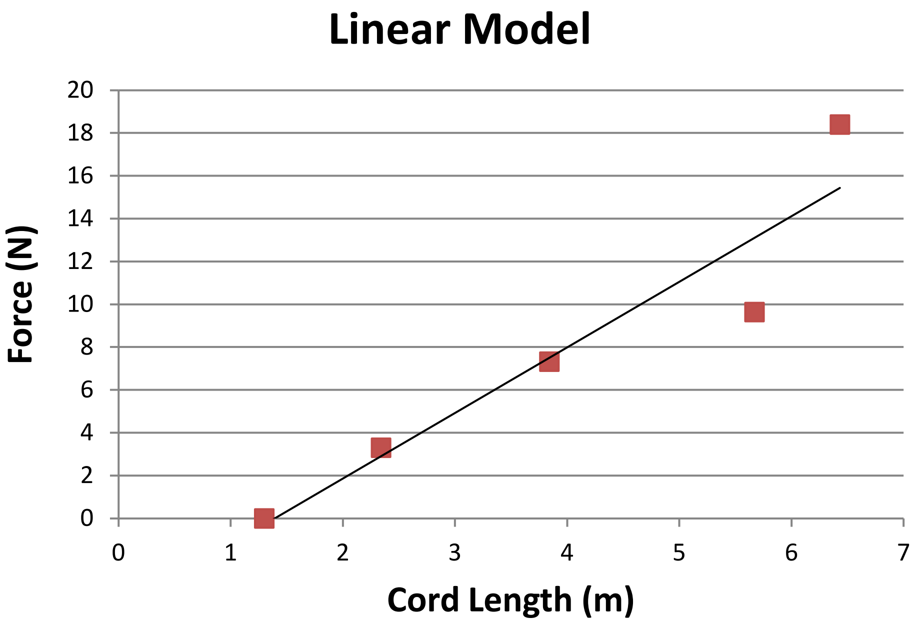
- Force Sensor
- Motion Sensor
- Data Rate: 5 Hz
- 6 m limit on L

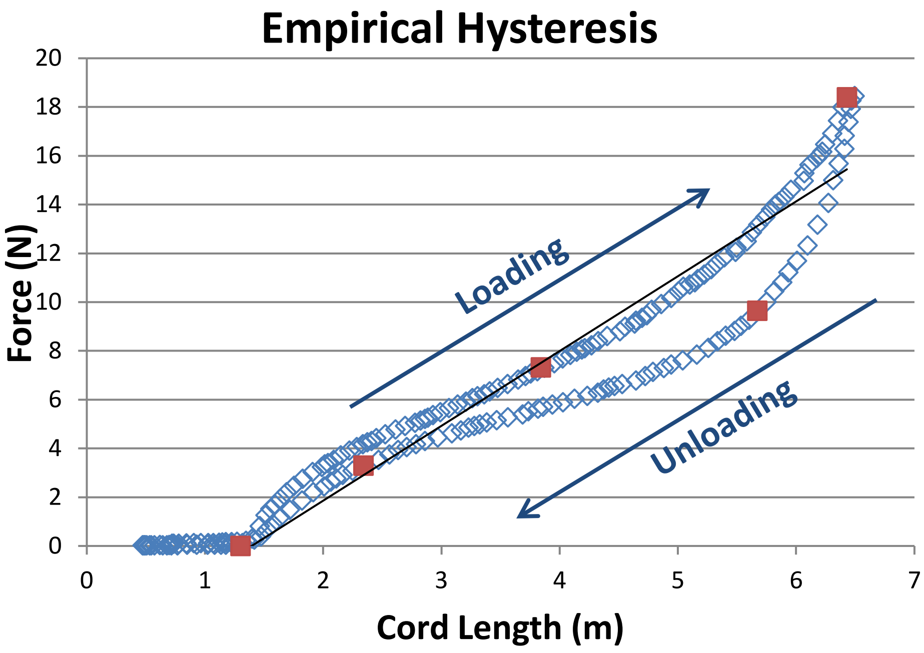
Single or Double Bands

- Force Sensor
- Rotary Motion Sensor + Pulley System & Counter Weights
- 2 m limit on L

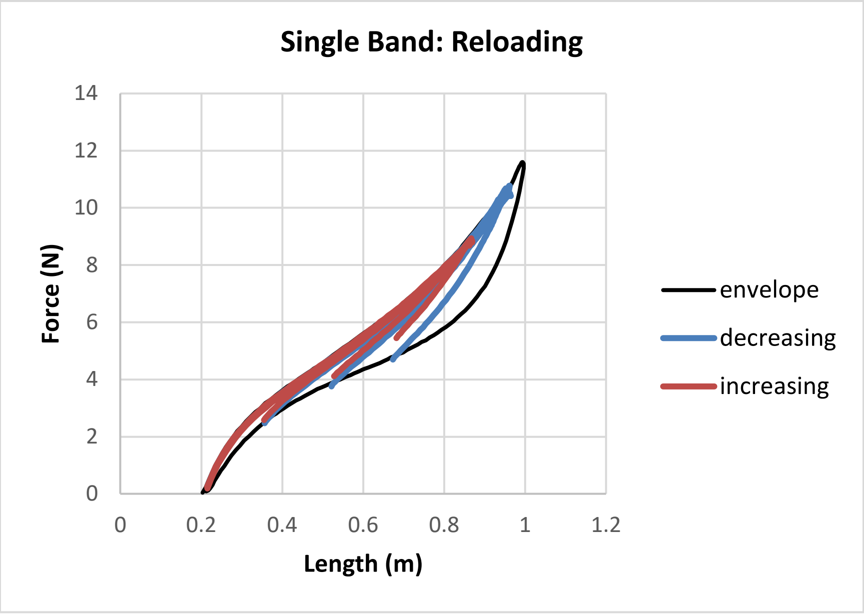
Modelling the Cord

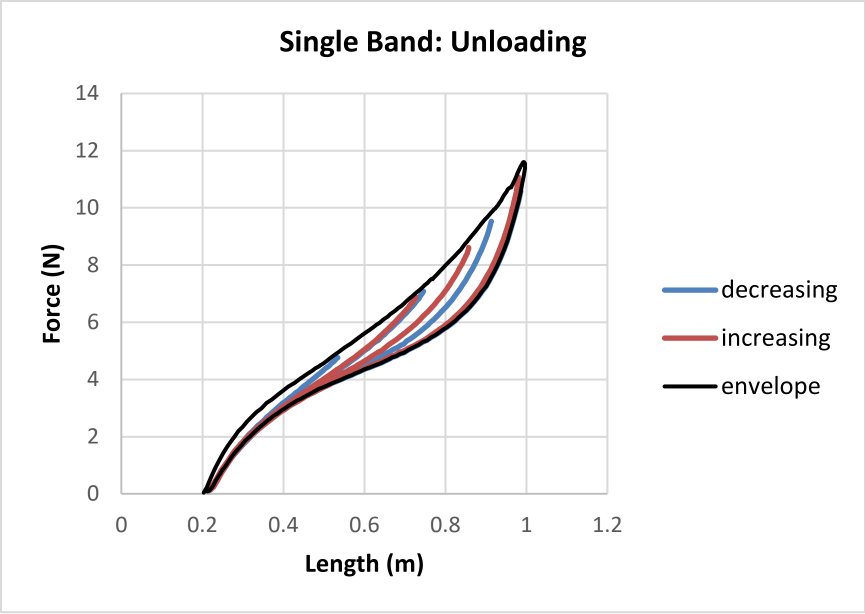
Force vs. Length





Single Band Hysteresis





Force Model: for strictly (un)loading process as a simple Function that approaches envelope at endpoints

Simulation Hysteresis Model

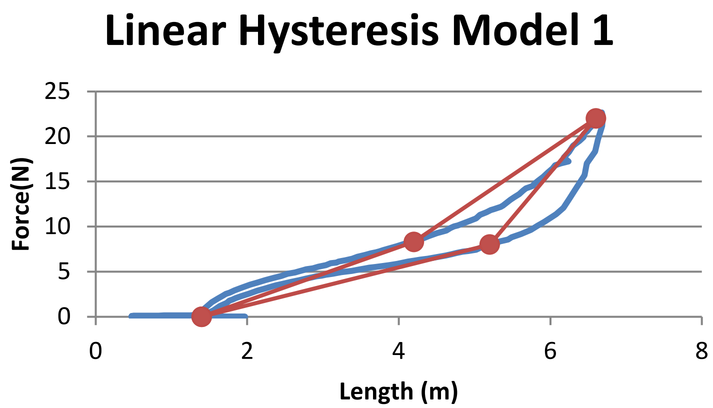
- Hysteresis Envelope
  - Loading and Unloading functions $F_L(x), F_U(x)$
- Rate Independent, Simple Model:
  - current state (Force and stretch)
  - change in stretch (loading vs. unloading)
  - For strictly loading/unloading process:
    - Force is a simple Function that approaches envelope at endpoints, determined by seed point (F_i, x_i)

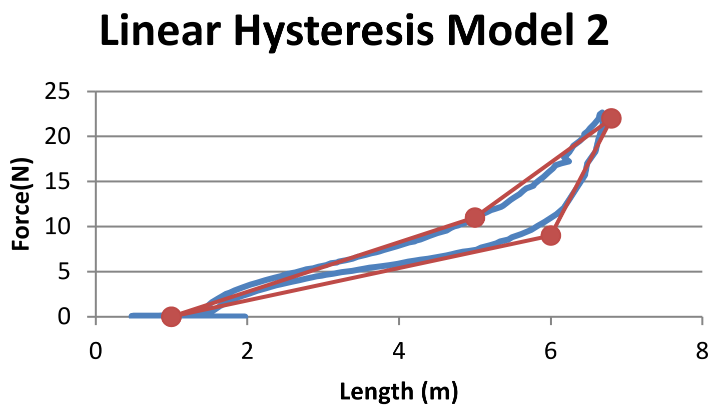
$$f_L(x, x_i, F_i) = F_L(x) - (F_L(x_i) - F_i) * \left|\frac{x - x_{max}}{x_i - x_{max}}\right|^{p_L} \;;\; p_L = 1$$

$$f_U(x, x_i, F_i) = F_U(x) + (F_U(x_i) - F_i) * \left|\frac{x - L_0}{x_i - L_0}\right|^{p_U} \;;\; p_U = 2.5$$

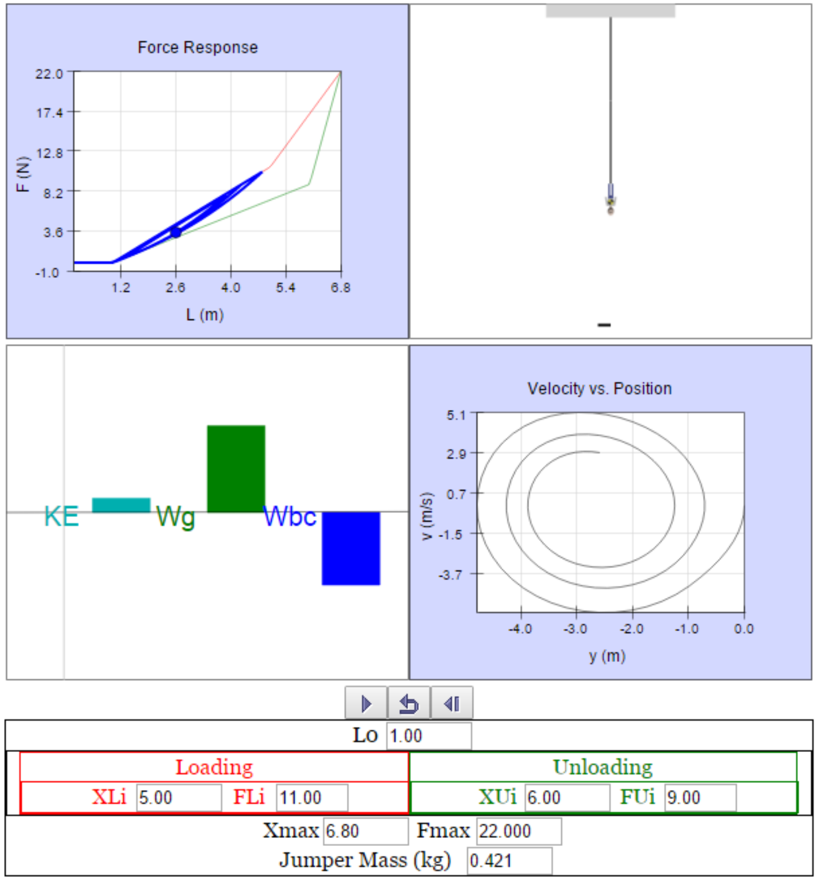
Hysteresis Envelope

- Measuring Envelope
  - Loading and Unloading functions $F_L(x), F_U(x)$
  - (Simple) Envelope parameterization
  - Linear Segments

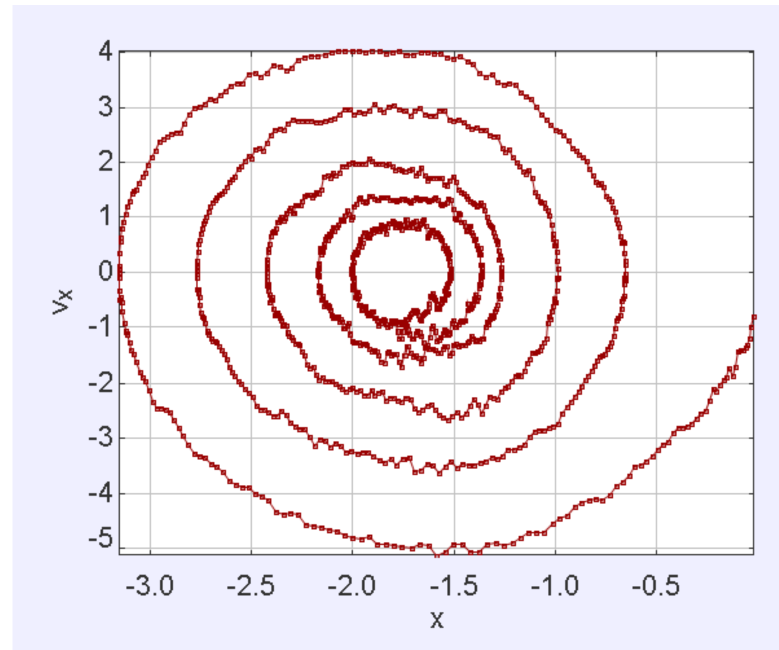




Simulation vs Video Analysis



EJSS Model



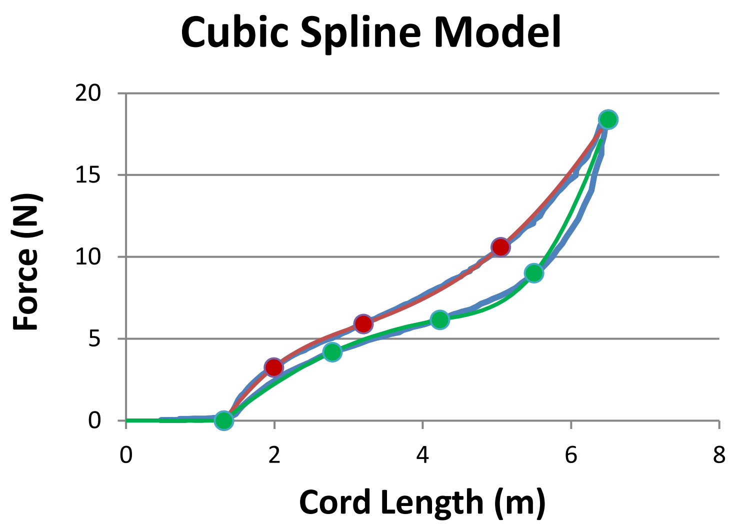
Tracker Video Analysis

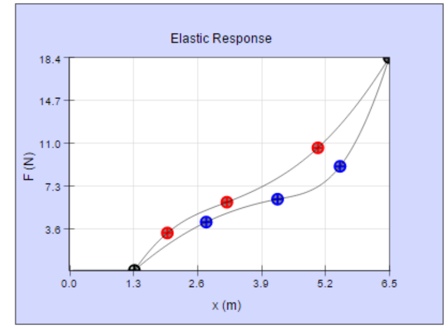
- 120 fps video

Issues

- Sensor Calibration (Force Sensor, Motion Sensor)
- Model Fidelity

Next Steps: Improved Envelope





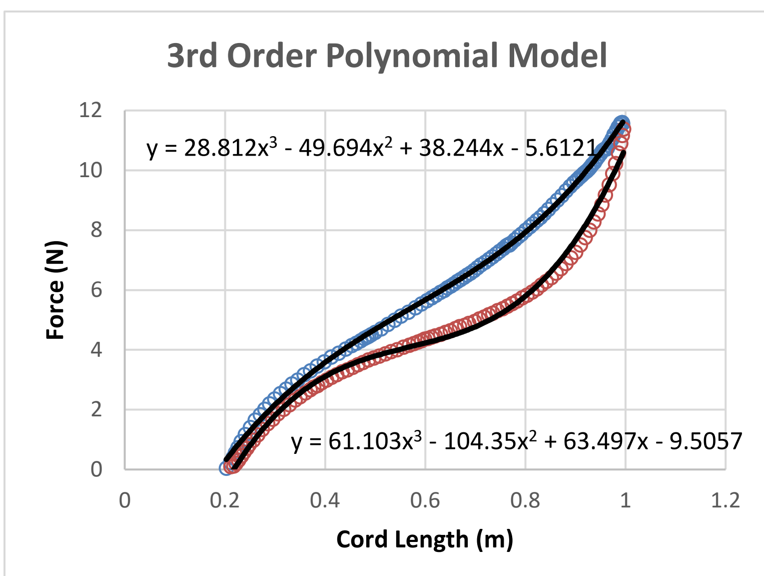
Elastic Data

Lo 1.32

Loading				Unloading			
XL1	1.99	FL1	3.24	XU1	2.78	FU1	4.18
XL2	3.20	FL2	5.90	XU2	4.23	FU2	6.16
XL3	5.05	FL3	10.60	XU2	5.50	FU3	9.00

Xmax 6.50 Fmax 18.400

update spline



Finale

- Hysteresis is “easy to measure” & relevant to fun activities
- Weight that statically doubles length will dynamically approach $L = 4 \times L_0$
- Simple Model can replicate some features
- Issues with implementation
  - Sensor Calibration (Force Sensor, Motion Sensor)
  - Model Fidelity
  - Temperature
- Model will be submitted to OpenSourcePhysics.org
- Thanks: Ziwei Yang and Jonathan Dahl

Single Band Response